Floating Wire Simulation of the Trajectory of a Charged Particle in a Magnetic Field

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1 Problem

Show that the trajectory of a charged particle in a magnetic field can be duplicated by that of a current-carrying wire held at rest under constant tension (by fixtures outside the field region. Deduce the current $I$ required in a wire of tension $T$ to match the trajectory of an proton of momentum $P$.

2 Solution

The equation of motion of a (relativistic) particle of charge $e$, mass $m$ and velocity $v$ in a magnetic field $B$ is (in MKSA units)

$$\frac{dP}{dt} = ev \times B.$$  \hspace{1cm} (1)

An increment $ds$ of arc length along the particle’s trajectory can be written

$$ds = v dt.$$ \hspace{1cm} (2)

Using this to replace $v$ in eq. (1), the particle’s trajectory can be described as

$$dP = e ds \times B.$$ \hspace{1cm} (3)

The momentum $P$ is along the trajectory, so we can write

$$P = P\hat{s}, \quad \text{and} \quad dP = Pd\hat{s},$$ \hspace{1cm} (4)

noting that the magnetic field changes the direction, but not the magnitude, of the momentum. Hence the equation of the trajectory is

$$d\hat{s} = \frac{e}{P}ds \times B.$$ \hspace{1cm} (5)

The equation for static equilibrium of a current-carrying wire under tension $T$ in the same magnetic field is

$$\sum F = 0 = T(s + d\hat{s}) - T\hat{s} + Ids \times B,$$ \hspace{1cm} (6)

or

$$d\hat{s} = -\frac{I}{T}ds \times B.$$ \hspace{1cm} (7)
The trajectory of the “floating” wire can be the same as that of the charged particle when

\[ I[A] = -\frac{eT[N]}{P[kg\cdot m/s]}, \tag{8} \]

We express this relation in practical units by noting that

\[ T[N] = 0.0098 \, T[gm], \tag{9} \]

when the tension is maintained by a weight \( T \) in grams attached to one end of the wire over a pulley, and that the momentum of the proton is

\[ P[kg\cdot m/s] = \frac{10^6 e}{c} P[MeV/c] = \frac{e}{300} P[MeV/c]. \tag{10} \]

Hence,

\[ I[A] = -\frac{2.94 \, T[gm]}{P[MeV/c]} \tag{11} \]